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New extension for Chen distribution based on [0, 1] Truncated Nadarajah-Haghighi-G family with two real data application

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Abstract
The [0, 1] Truncated Nadarajah-Haghighi Chen distribution, a novel probability model with four parameters, is introduced. The analysis considers significant statistical characteristics of the distribution, such as the Rényi entropy, moments, incomplete moments, moment-generating function, and order statistics of the quantile function. The parameters of the new model were determined using maximum likelihood estimators for population parameters. A simulation study is done to estimate the model parameters of the TNHCh distribution. In addition, two sets of data were proposed to evaluate the compatibility of the new distribution with other comparable distributions.

Keywords: Chen distribution, moments, quantile function, Lorenz curve, [0, 1] Truncated, MLE

1. Introduction
Various practical fields, such as engineering, banking, and medicine, rely extensively on modeling and analysis of longevity data. Various lifespan distributions have been employed to represent this type of data. The effectiveness of the methods employed in a statistical research is greatly influenced by the assumed probability model or distributions. As a result, significant effort has been devoted to developing comprehensive categories of standard probability distributions and relevant statistical methods. However, there are numerous severe problems where the actual data does not conform to any of the traditional or standard probability models. This study introduces a novel extension of the Rayleigh distribution known as the [0, 1] Truncated Nadarajah-Haghighi Chen distribution. The Chen distribution has been extensively investigated by numerous researchers. The study by Khan *et al.* (2015) [7] introduced the concept of the transmuted exponentiated Chen distribution. The Exponentiated Chen distribution was introduced by Dey *et al.* in 2017 [3]. The Gamma-Chen distribution was introduced by Reis *et al.* (2020) [10]. The paper titled "On the extended Chen distribution" was authored by Bhatti *et al.* in 2021 [2]. Tarvirdizade and Ahmadpour (2021) [12] proposed a novel extension of the Chen distribution and shown its utility in analyzing lifetime data. The unit-Chen distribution was introduced and examined by Korkmaz *et al.* (2022).

2. The New TNHCh Model
By using the ([0, 1] Truncated Nadarajah-Haghighi-G family) (Al-Habib *et al.*, 2023) [1] we get the cumulative distribution function and the probability density function of the suggestion distributions are as follows.

$$F(y)_{TNH-G} = \frac{1 - e^{1-\left(1+\beta M(y,\varphi)\right)^{\alpha}}}{1 - e^{1-(1+\beta)^{\alpha}}} \quad (1)$$

And

$$f(y)_{TNH-G} = \frac{ab\left(1 + bM(y,\varphi)\right)^{\alpha-1} e^{1-(1+\beta M(y,\varphi))^{\alpha}} m(y,\varphi)}{1 - e^{1-(1+\beta)^{\alpha}}} \quad (2)$$

where $M(y,\varphi)$, $m(y,\varphi)$ Represent the CDF and PDF for the distribution to be expanded distribution. Therefore, for the Chen distribution we have (Thanh and Briš, 2021) [13]:

$$M(y,\varphi) = 1 - e^{a\left(1-e^{y^b}\right)};\ y > 0, a, b > 0 \quad (3)$$

$$m(y,\varphi) = abx^{b-1} e^{a\left(1-e^{y^b}\right)+y^b};\ y > 0, a, b > 0 \quad (4)$$

Now by insert (3) , (4) in (1), (2) respectively we get that

$$F(y) = \Delta\left[1 - e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}}\right] \quad (5)$$

And

$$f(y) = \Delta\alpha\beta abx^{b-1}\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha-1} e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} e^{a\left(1-e^{y^b}\right)+y^b} \quad (6)$$

Where $\Delta = \frac{1}{1 - e^{1-(1+\beta)^{\alpha}}}$

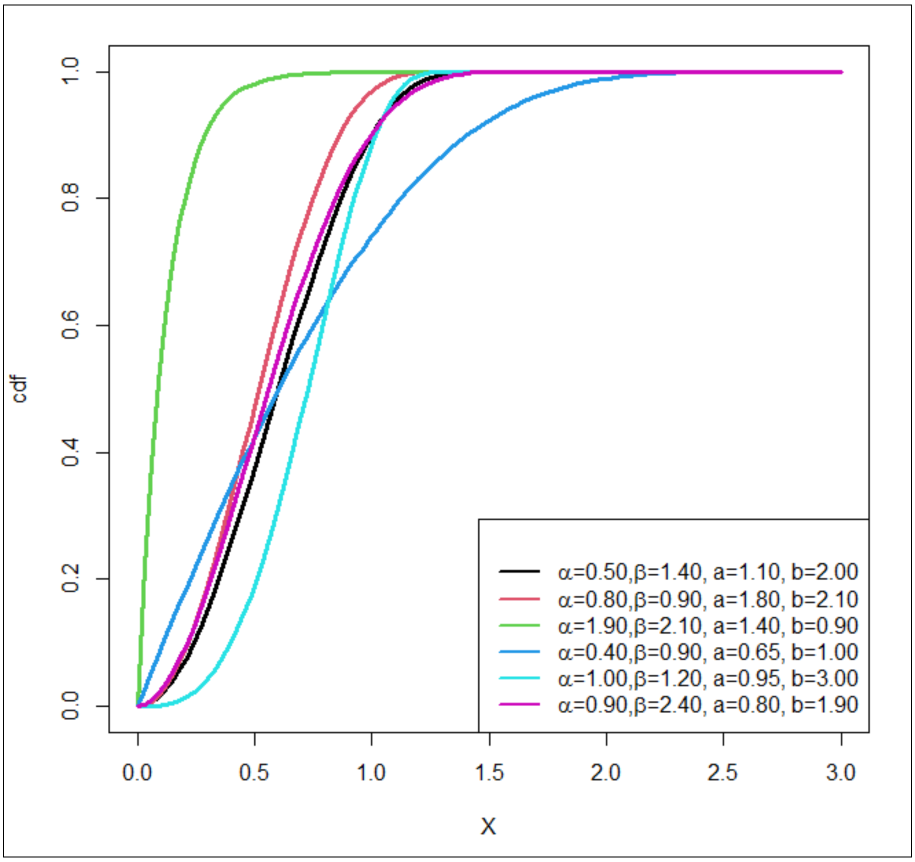

Fig 1: CDF shapes for the new distribution

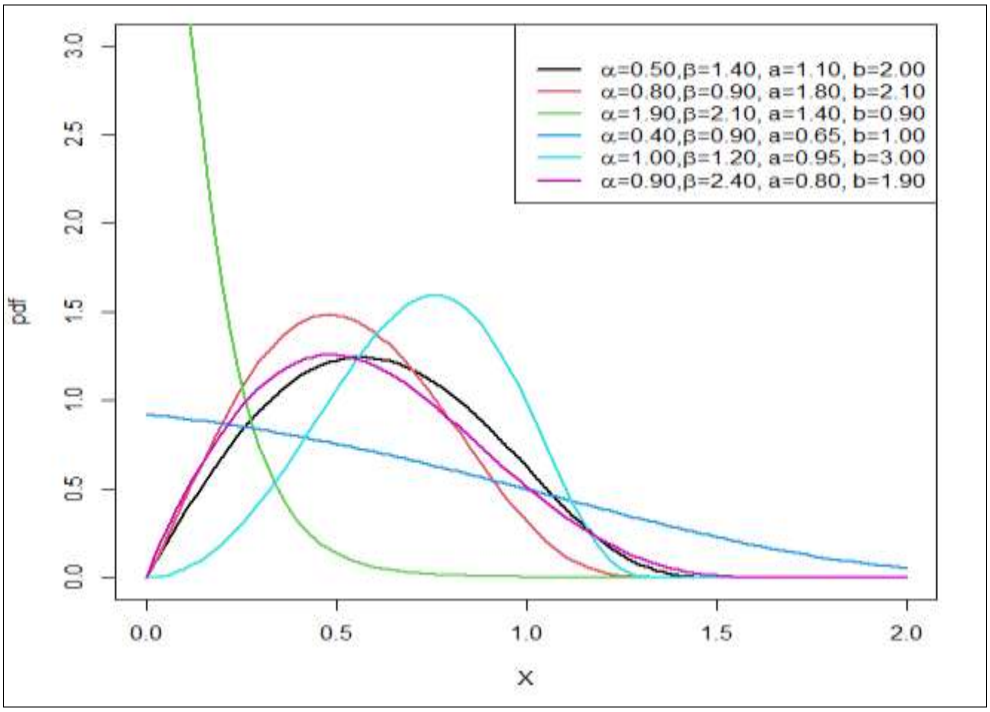

Fig 2: PDF shapes for the new distribution

Survival Function

A key idea in probability theory and statistics, especially in survival analysis, is the survival function. It shows the likelihood that a specific failing or event won't happen by a given deadline. Put more simply, it provides the likelihood that a person or system will last past a specific amount of time. The survival function for the TNHCh distribution can be written as follow (Khaleel 2022) [5].

$$S(y) = \frac{e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} - e^{1-(1+\beta)^{\alpha}}}{1 - e^{1-(1+\beta)^{\alpha}}} \tag{7}$$

Hazard Function

Another key idea in reliability theory and survival analysis is the hazard function. If the subject or system has lasted until a particular time, it indicates the instantaneous rate at which an event occurs.Put more simply, the hazard function expresses the likelihood that an event will transpire in the upcoming moment, presuming that it hasn't happened yet. Given the survival up to that time, it aids in determining the danger or likelihood of an event occurring at any given moment. Hazard function for [0,1]TNHCh distribution is given by :

$$h(y) = \frac{\alpha\beta a b x^{b-1}\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha-1} e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} e^{a\left(1-e^{y^b}\right)+y^b}}{e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} - e^{1-(1+\beta)^{\alpha}}} \tag{8}$$

3. Expansion Probability Density Function for the new model

A linear representation of the probability density function was suggested in this section. Give the PDF's linear representations in this section: by take eq. (6) and reduce it. By [12] we get that the expansion of pdf for the [0,1] TNH-G family is given by:

$$f(y) = \Omega_{j,k,m}\; m(y,\xi)\left(M(y,\xi)\right)^{m(\alpha k+\beta-1)}$$

where

$$\Omega_{j,k,m} = \frac{\alpha\beta}{1-e^{1-[1+\beta]^{\alpha}}} \sum_{j=0}^{\infty}\sum_{k=0}^{j}\sum_{m=0}^{\infty} \binom{j}{k} \frac{(-1)^k}{j!} \binom{\alpha k+\alpha-1}{m} \beta^m$$

Now by (3), (4) the linear representation for pdf is given by

$$f(y) = \Omega_{j,k,m}\ aby^{b-1}e^{y^b}e^{a\left(1-e^{y^b}\right)}\left(1-e^{a\left(1-e^{y^b}\right)}\right)^{m(\alpha k+\beta-1)}$$

by use the generalized binomial theorem formula

$$(1-Z)^j = \sum_{s=0}^{\infty}\binom{j}{s}(-1)^s\,(Z)^s$$

Then

$$\left(1-e^{a\left(1-e^{y^b}\right)}\right)^{m(\alpha k+\beta-1)} = \sum_{s=0}^{\infty}\binom{m(\alpha k+\beta-1)}{s}(-1)^k\,e^{as\left(1-e^{y^b}\right)}$$

So that

$$f(y) = \Omega_{j,k,m}\ abx^{b-1}e^{y^b}\sum_{s=0}^{\infty}\binom{m(\alpha k+\beta-1)}{s}(-1)^k\,e^{a(s+1)\left(1-e^{y^b}\right)}$$

by use the expansion exponential formula (Habib *et al*., 2023) [4].

$e^u = \sum_{j=0}^{\infty}\frac{u^j}{j!}$, we have gotten

$$e^{a(s+1)\left(1-e^{y^b}\right)} = \sum_{d=0}^{\infty}\frac{(a(s+1))^j}{j!}\left(1-e^{y^b}\right)^d$$

Hence

$$f(y) = \Omega_{j,k,m}\ abx^{b-1}e^{y^b}\sum_{s=0}^{\infty}\sum_{d=0}^{\infty}\frac{(a(s+1))^j}{j!}\binom{m(\alpha k+\beta-1)}{s}(-1)^k\left(1-e^{y^b}\right)^d$$

Similarly by generalized binomial theorem have that

$$\left(1-e^{y^b}\right)^d = \sum_{z=0}^{\infty}\binom{d}{z}(-1)^z\,e^{zy^b}$$

Then

$$f(y) = \Omega_{j,k,m}\ ab\sum_{s,z,d=0}^{\infty}\binom{m(\alpha k+\beta-1)}{s}x^{b-1}e^{(z+1)y^b}\binom{d}{z}(-1)^{z+k}\frac{(a(s+1))^j}{j!}$$

Let

$$\otimes_{j,k,m,s,z,d} = \Omega_{j,k,m}\ ab\sum_{s,z,d=0}^{\infty}\binom{m(\alpha k+\beta-1)}{s}\binom{d}{z}(-1)^{z+k}\frac{(a(s+1))^j}{j!}$$

Then

$$f(y) = \otimes_{j,k,m,s,z,d}\ y^{b-1}e^{(z+1)y^b} \tag{9}$$

4. Quantile function

A statistical concept called the quantile function, or inverse cumulative distribution function, is used to calculate the value at which a specific percentage of a probability distribution resides below that value. The TNHCh quantile function can be obtained By reversing the equation defined in (5) as follows:

$$u = \Delta \left[1 - e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} \right]$$

$$1 - e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} = \frac{u}{\Delta}$$

$$1 - \frac{u}{\Delta} = e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}}$$

$$1 - \ln\left(1 - \frac{u}{\Delta}\right) = \left(1 + \beta\left(1 - e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}$$

$$\left(1 - e^{a\left(1-e^{y^b}\right)}\right) = \frac{1}{\beta}\left[1 - \ln\left(1 - \frac{u}{\Delta}\right)\right]^{\frac{1}{\alpha}} - 1$$

$$\left(1 - \frac{1}{\beta}\left[1 - \ln\left(1 - \frac{u}{\Delta}\right)\right]^{\frac{1}{\alpha}} - 1\right) = e^{a\left(1-e^{y^b}\right)}$$

$$e^{y^b} = \left(1 - \frac{1}{a} ln\left\{\left(1 - \frac{1}{\beta}\left[1 - ln\left(1 - \frac{u}{\Delta}\right)\right]^{\frac{1}{\alpha}} - 1\right)\right\}\right)$$

$$y = \left\{ ln\left[\left(1 - \frac{1}{a} ln\left\{\left(1 - \frac{1}{\beta}\left[1 - ln\left(1 - \frac{u}{\Delta}\right)\right]^{\frac{1}{\alpha}} - 1\right)\right\}\right)\right]\right\}^{\frac{1}{b}} \quad (10)$$

The average of the TNHCh random variable can be obtained by setting the parameter u equal to 0.5, starting with a value of 10. Quantile function is used as the basis for alternative metrics of skewness and kurtosis. This is particularly useful when the quantile function of the distribution is in a closed form. The Galton's (Galton, 1883) quantile based measure of skewness and the Moors' (Moors, 1988) quantile based measure of kurtosis are respectively given by the following forms where skewness S_1 and kurtosis S_2 as:

$$S_1 = \frac{Q(0.75) - 2Q(0.5) + Q(0.25)}{Q(0.75) - Q(0.25)}$$

$$S_2 = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(0.75) - Q(0.25)}$$

5. Moments

The mean, variance, skewness, and kurtosis of a probability distribution are all determined in large part by moments. So the r^{th} moment of the TNHCh distribution is determined by:

Since the r^{th} moment of a random variable X defined as

$$\mu_r = \int_{-\infty}^{\infty} y^r f(y)\, dy$$

Where $f(y)$ given in (9) so that.

$$\mu_r = \otimes_{j,k,m,s,z,d} \int_{0}^{\infty} y^{r+b-1} e^{(z+1)y^b}\, dy$$

Then

$$\mu_r = \otimes_{j,k,m,s,z,d} \frac{(-1)^{\frac{r}{b}+1}\Gamma(\frac{r}{b}+1)}{(k+1)^{\frac{r}{b}+1}}$$

The following table makes it evident that when the fourth value rises and the first, second, and third values remain constant, the first and fourth moments rise along with a decline in the skewness and kurtosis values. Additionally, it is evident that there is a distinct increase in the values of skewness and kurtosis and a decrease in the values of moments and anisotropy when the values of the first, second, and fourth parameters remain constant while the value of the third parameter increases. This also holds true for the second value's growth and the stability of the parameters' first, third, and fourth values.

Table 1: The first fourth moments, variance, skewness and kurtosis for the TNHCh distribution.

α	β	a	b	μ_1	μ_2	μ_3	μ_4	$Var(x)$	Sk	Ku
Values of parameters				**Values of properties**						
2	5	3	2.4	0.199	0.046	0.011	0.003	0.0064	1.11495	1.41777
2	5	3	3.4	0.314	0.107	0.039	0.014	0.0084	1.11427	1.22281
2	5	1	2.4	0.310	0.111	0.044	0.018	0.0149	1.18978	1.46092
2	5	6	2.4	0.149	0.026	0.005	0.001	0.0038	1.19264	1.47929
2	0.5	3	2.4	0.472	0.262	0.163	0.109	0.03922	1.21545	1.5879
2	1.5	3	2.4	0.343	0.140	0.065	0.033	0.02235	1.24086	1.68367
1	5	3	2.4	0.299	0.110	0.047	0.023	0.0206	1.28828	1.90083
3	5	3	2.4	0.162	0.030	0.006	0.001	0.00376	1.1547	1.11111

6. Moment Generating Function

A useful tool in probability theory and statistics that describes a probability distribution is the moment generating function (MGF). It gives an approach to obtain a probability distribution's moments, making it possible to compute the distribution's mean, variance, skewness, and other moments. The moment generating functions for TNHCh distribution is determined by:
Given that the random variable Y's moment generating function is defined as follows

$$M_Y(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_{-\infty}^{\infty} y^r\, f(y) dy$$

Where $f(y)$ was defined in (9) we have get

$$M_Y(t) = \otimes_{j,k,m,s,z,d} \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{(-1)^{\frac{r}{b}+1}\Gamma(\frac{r}{b}+1)}{(k+1)^{\frac{r}{b}+1}}$$

7. Characteristic Function

In probability theory and statistics, the characteristic function is an essential idea that offers a singular means of fully characterizing a probability distribution. The probability distribution of a random variable is defined by this mathematical function. The characteristic function of TNHCh distribution can be get from the following form

$$Q_Y(t) = \sum_{z=0}^{\infty} \frac{(it)^z}{z!} \int_0^{\infty} y^z f(y) dy$$

where $f(y)$ was defined in (9) we have get

$$Q_Y(t) = \otimes_{j,k,m,s,z,d} \sum_{z=0}^{\infty} \frac{(it)^z}{z!} \frac{(-1)^{\frac{r}{b}+1} \Gamma(\frac{r}{b}+1)}{(k+1)^{\frac{r}{b}+1}}$$

8. Incomplete Moments

The fundamental idea of incomplete moments (IM) serves as the basis for the inequality measures in distribution investigations; so, the incomplete moments of the TNHCh distribution are defined as. Considering that a random variable Y defined by its incomplete moments

$$M_r(x) = \int_{-\infty}^{x} y^r f(y) dy$$

Where $f(y)$ given in (9) we have get:

$$M_r(x) = \otimes_{j,k,m,s,z,d} \int_{0}^{x} y^{r+b-1} e^{(z+1)y^b} \, dy$$

9. Lorenz Curve

Economists and policymakers can better understand income or wealth distribution gaps in a society by using the Lorenz curve. It offers a means of comparing and evaluating the degree of inequality. The Lorenz curve for the new distribution can be obtained from the following form:

$$L_F(x) = \frac{1}{\mu} \int_{-\infty}^{x} y f(y) dy$$

So that where $f(y)$ defined in (9) we get the Lorenz curve as follows:

$$L_F(x) = \frac{1}{\mu} \otimes_{j,k,m,s,z,d} \int_{0}^{x} y^b e^{(z+1)y^b} \, dy$$

10. Bonferroni Curve

For a random variable Y The Bonferroni Curve can be defined as $B_F(x) = \frac{L_F(x)}{F(x)}$ now where The Lorenz curve for the new distribution and $F(x)$ was defined in (5) with respect to x so that:

$$B_F(x) = \frac{1}{\Delta\mu \left[1 - e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{x^b}\right)}\right)\right)^{\alpha}}\right]} \otimes_{j,k,m,s,z,d} \int_{0}^{x} y^b e^{(z+1)y^b} \, dy$$

11. Order Statistic

Given a [0, 1] TNHCh distribution with Y_1, Y_2, Y_3,... and CDF and PDF specified in (5) and (6), respectively, let Y_(1:m), Y_(2:m), Y_(3:m),... and Y_(m:m) be the order statistics derived from this sample. Next, the following is the probability density function of the p^th order statistic from the [0, 1] TNHCh distribution:

The PDF of order statistic with order $p, Y_{p;m}$ is given by the form:

$$f_{k:m}(x) = \frac{m!}{(k-1)!(m-k)!} [F(y)]^{k-1}[1 - F(y)]^{m-k} f(y)$$

$$= \sum_{s=0}^{m-k} D(-1)^s \binom{m-k}{s} [F(y)]^{k+s-1} \; f(y)$$

Where $D = \frac{m!}{(k-1)!(m-k)!}$

Then

$$f_{k:n}(x) = \sum_{s=0}^{m-k} D(-1)^s \binom{m-k}{s} \left[\Delta \left[1 - e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} \right] \right]^{k+s-1} *$$

$$\Delta\alpha\beta ab x^{b-1} \left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha-1} e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} e^{a\left(1-e^{y^b}\right)+y^b}$$

For k=1, the lowest order statistic (also known as the least value function) is obtained, whereas for k=m, the greatest order statistic (also known as the huge value function) is obtained:

12. Mean Residual Life

The mean residual life (MRL) is a frequently employed concept in the fields of survival analysis and reliability theory. It denotes the mean duration of life or functionality that an individual or system might expect to have, provided that it has already endured until a specific moment in time. The mean residual life of the TNHCh distribution can be obtained from the following formula:

$$\bar{M}(x) = \frac{1}{S(x)}\left(\int_{x}^{\infty} y f(y) dy\right) - x$$

now substituted $f(y)$ and $S(x)$ as defined in (9) and (7) respectively get that:

$$\bar{M}(x) = \frac{1 - e^{1-(1+\beta)^{\alpha}}}{e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{x^b}\right)}\right)\right)^{\alpha}} - e^{1-(1+\beta)^{\alpha}}} \left(\otimes_{j,k,m,s,z,d} \int_{x}^{\infty} y^b e^{(z+1)y^b} dy\right) - x$$

13. Maximum Likelihood Method

Let's assume that y_1, y_2, ..., y_n is a random sample of size n from the TNHCh distribution. The equivalent likelihood function is provided in Mead *et al.* (2019) [9] and Khaleel *et al.* (2020) [6]. For further details, please refer to these sources. Empty text.

$$L(\emptyset) = \prod_{i=1}^{n} f(y_i, \emptyset)$$

$$L(\emptyset) = \prod_{i=1}^{n} \frac{\alpha\beta ab y^{b-1} \left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha-1} e^{1-\left(1+\beta\left(1-e^{a\left(1-e^{y^b}\right)}\right)\right)^{\alpha}} e^{a\left(1-e^{y^b}\right)+y^b}}{1 - e^{1-(1+\beta)^{\alpha}}}$$

$$L(\emptyset) = (\alpha\beta ab)^n + (b-)\sum_{i=1}^{n} \log(y_i) + (\alpha - 1)\sum_{i=1}^{n} \log\left\{\left(1+\beta\left(1-e^{a\left(1-e^{y_i^b}\right)}\right)\right)\right\} + \sum_{i=1}^{n}\left(1-\left(1+\beta\left(1-e^{a\left(1-e^{y_i^b}\right)}\right)\right)^{\alpha}\right) + \sum_{i=1}^{n}\left(a\left(1-e^{y_i^b}\right)+y_i^b\right) - n\,log\{1 - e^{1-(1+\beta)^{\alpha}}\}$$

Application: To show that our suggested distribution fits the data better than alternative distributions, we have fitted the [0, 1] TNHCh distribution to an actual dataset in this section. The R statistical software was used to perform the calculations. To find the best model fit, we used a variety of statistical measures, including the likelihood ratio test (-l), the Akaike information criterion (AIC), the Bayesian information criterion (BIC), and the Hannan-Quinn information criterion (HQIC). The Chen distribution, Gompertez Chen (GoR), Weibull Chen (WeCh), Kumaraswamy Chen (Kuch), Beta Chen (BeR), and Exponential Generalized Chen (EGCh) were contrasted with our suggested model.

Data set:- complete data for failure times of 20 components (Lai *et al*., 2003) [8] and (Srivastava, 2019) [11].:
0.481, 1.196, 1.438, 1.797, 1.811, 1.831, 1.885, 2.104, 2.133, 2.144, 2.282, 2.322, 2.334, 2.341, 2.428, 2.447, 2.511, 2.593, 2.715, 3.218.

- The suggested [0,1]TNHCh distribution has the biggest p-value and the smallest values for K-S, A, and W, according to the data shown in Table 1. This implies that, in comparison to the other competing distributions, the suggested distribution fits the data better.
- Table 2 shows that, in comparison to the other distributions' log-likelihood values, the suggested [0,1] TNHCh distribution's log-likelihood value is less. Furthermore, the suggested distribution's AIC value is lower than the AIC values of the other distributions. This suggests that the suggested distribution fits these datasets better.

Table 1: The K-S, A, W and p-value for the data set.

A				
[0, 1] TNHCh	0.1101	0.9465	0.0440	0.3049
BCh	0.1237	0.8829	0.0633	0.4191
KuCh	0.1238	0.8826	0.0629	0.4161
EGCh	0.1297	0.8472	0.0640	0.4453
WeCh	0.1297	0.8470	0.0654	0.4341
GoCh	0.1565	0.6550	0.0739	0.5129
Ch	0.1376	0.7943	0.0643	0.4478

Table 2: The values of -2l, AIC, CAIC, BIC, HQIC

l						
[0,1] TNHCH	$\hat{\alpha}$ =7.424 $\hat{\beta}$ = 0.467 $\hat{a}$ = 0.014 $\hat{b}$ = 1.440	15.79	37.26	37.97	39.26	37.65
BCH	$\hat{\alpha}$ =1.495 $\hat{\beta}$ = 0.778 $\hat{a}$ =0.096 $\hat{b}$ =1.248	16.36	40.73	43.40	44.71	41.51
KuCH	$\hat{\alpha}$ =1.553 $\hat{\beta}$ = 2.090 $\hat{a}$ =0.051 $\hat{b}$ =1.195	16.35	40.71	43.38	44.69	41.49
EGCH	$\hat{\alpha}$ = 0.160 $\hat{\beta}$ = 1.055 $\hat{a}$ =0.257 $\hat{b}$ =1.359	16.56	41.13	43.80	45.11	41.91
WeCH	$\hat{\alpha}$ =1.766 $\hat{\beta}$ =0.492 $\hat{a}$ =0.052 $\hat{b}$ =1.000	16.45	40.90	43.57	44.88	41.68
GoCH	$\hat{\alpha}$ =0.926 $\hat{\beta}$ = 0.164 $\hat{a}$ =0.054 $\hat{b}$ =1.257	17.06	42.14	44.81	46.12	42.92
CH	$\hat{\alpha}$ =0.041	16.63	39	42	43	40

	$\hat{\beta}$ = 1.360					

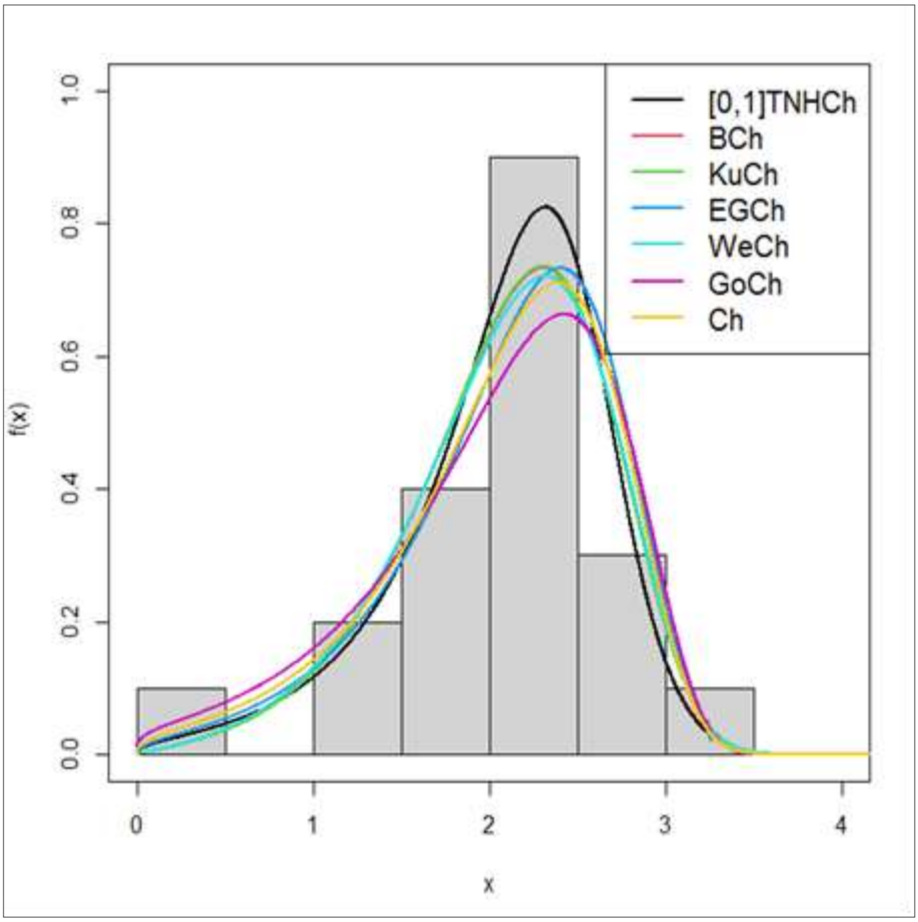

Fig 3: Estimated densities of the models for data set.

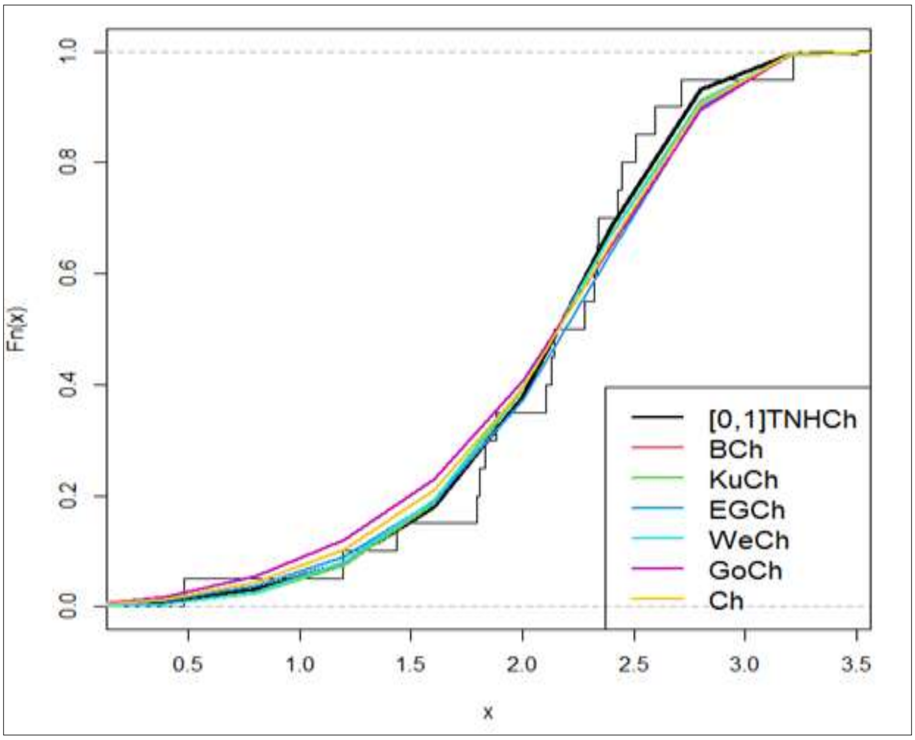

Fig 3: Empirical, fitted cdf of the data set.

As seen in Figures 3 and 4, the fitted density of the TNHCh distribution outperforms the distributions of BCh, KuCh, EGCh, WeCh, GoCh, and Chen, and it nearly matches the actual histogram.

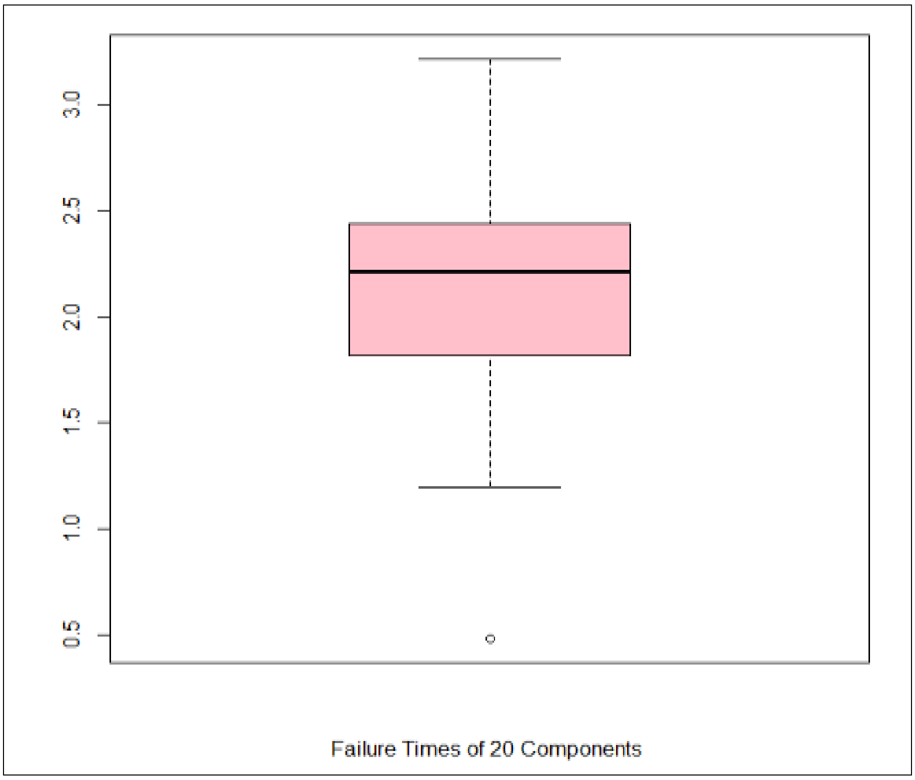

Fig 4: Box plot for the data set.

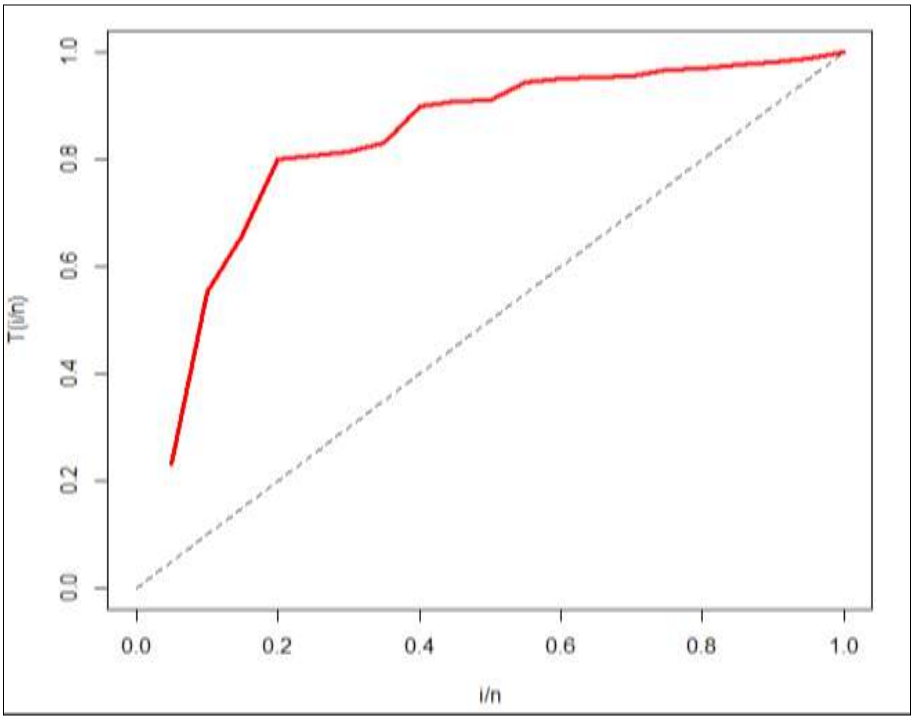

Fig 5: TTT plot for the data set.

14. Conclusion

This study presents a novel model, known as the [0, 1] Truncated Nadarajah-Haghighi Chen distribution, which expands the use of the Chen distribution to data with a true support range. The expanded version of a typical distribution provides greater flexibility for modelling, which serves as a clear rationale for its generalization. We derive expansions for both the moments and the function that generates the moments. Parameter estimation is approached using the maximum likelihood method. The [0, 1] TNHCh distribution can be effectively applied to real data, resulting in better fits compared to the Chen distribution.

15. References

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