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# Population dynamics of unemployment in a recessed economy: The Nigerian perspective

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**Abstract**
This work builds and analyzed a new mathematical model that investigates the impact of economic decline on the dynamics of unemployment during economic recession (which according to this study is for 12 months). Mathematical analysis reveals that the model is both locally asymptotically stable but globally asymptotically stable with a condition. Theoretical analysis shows that a decline in economic activities affects the rate of unemployment negatively. We simulated our models with two scenarios; the rate of economic decline was taken to be 0.0158 (1.58%) and 0.00158 (0.158%). The results revealed that the rate of economic decline determines the rate of negative effect on unemployment as well as the rates of lay-offs in public and private sectors. We recommended the need for individuals to become self-employed as part of government intervention strategy during this period.

**Keywords:** Population, unemployment, recession, self-employment, Nigeria

## 1. Introduction

The problem of unemployment has posed a great challenge to many countries (both developed and developing). It is an extremely severe social and economic problem born from the difference between demand and supply of the labor market and sometimes emphasized by population growth. The incidence of unemployment is usually deep and widespread. Generally, unemployment is a precarious social situation since a portion of the population normally struggles to maintain a minimum welfare and consumption level. The International Labor Organization (ILO, 2007) [2] defined unemployment as the number of economically active population who is without work but available and seeking work, including people who have lost their job and those who have voluntarily left work. The National Bureau of Statistics (2009) [7] sees the unemployed individuals with no work but are looking for work at the time of any study. This implies that unemployment describes the condition of people without jobs.

In Nigeria, the unemployment rate is the criteria for measuring the total number of people that are actively in search of one form of job or the other. It is recorded as a percentage of the labor force obtainable across the nation. According to the National Bureau of Statistics (2017) [8], the rate of unemployment in Nigeria had been on a consistent rise and in recent times, it rose for the seventh straight month to the highest it had been in recent years. The Nigerian unemployment rate was at 13.9 percent in the third quarter of 2016. The recent economic downturn (economic recession) is said to be responsible for this consistent rise in the Nigerian unemployment rate. Most companies sacked their workers and they left these workers adrift without any job. Many of these companies claimed that doing business in Nigeria was becoming unprofitable and had to leave for other countries.

Several studies have been conducted to model unemployment in the human population. Misra and Singh (2011) [3] proposed a non-linear mathematical model of unemployment based on Nikolopoulos and Tzanetis (2003) [6] who suggested a model for housing allocation of a homeless population due to a natural disaster. Misra and Singh (2011) [3] used a system of ordinary differential equations with the following three variables; unemployed population, employed individuals and temporary workers. They analyzed the equilibrium of the model using the stability theory and performed a few numerical simulations. They concluded that the unemployment battle may need immediate measures as unemployment rate may rise quickly and if high values are reached, then it might be really difficult to overcome a much bigger problem in the future.

Misra and Singh further developed their work in 2013 by replacing the temporal employed variable by newly created vacancies with a delayed future. Harding and Neamtu (2016) [1] followed the ideas of Misra and Singh (2013) [4], extending further the previous efforts by presenting an unemployment model where job search is open to both native and migrant workers. They considered two policy approaches: the first that aims at reducing unemployment by observing both past values of unemployment and migration; the second is by considering previously mentioned bibliography. In the work of Munoli and Gani (2016) [5], they sought to define an optimal control policy of unemployment means through two possible measures: government policies focused at providing jobs directly to unemployed persons and government policies aimed at creating new vacancies.

In this paper, we attempt to model unemployment through non-linear ordinary differential equations considering the Nigerian reality. The following assumptions were made:

i. All entrants to the unemployed class are fully qualified and competent to work in government-owned or private-owned institutions or become self-employed.
ii. Some of the unemployed persons may be employed in government-owned offices or in private-owned institutions.
iii. The rate of migration of unemployed persons is proportional to the number of unemployed persons.
iv. The rates of migration from the unemployed class to the employment class in government and private-owned institutions, self-employed class and the retired class is affected by economic downturn (recession).
v. The total number of available vacancies is limited.
vi. Individuals who quit their jobs from the government and private-owned institutions to start-up their businesses are assumed to be unemployed upon leaving their jobs before the start-up.
vii. The duration of recession in this study is 12 months (four consecutive quarters).

## 2. Materials and Methods

### 2.1 Model Formulation

Let the total population of individuals to employable age and status be denoted by $N_H(t)$. This entire population is divided into five classes; unemployed individuals (U), employed individuals in private sectors ($E_p$), individuals employed in public sectors ($E_G$), retired individuals (R), and self-employed individuals (S). Thus we have:

$$N_H(t) = u(t) + E_p(t) + E_G(t) + R(t) + S(t) \quad (1)$$

We assume that within the period of recession, there is a constant influx of unemployed individuals into the population denoted by ^. Individuals in the unemployed class are hired in private sector (public sector) at a rate $a_1(a_2)$. The parameter also depends on the available vacancies in these sectors. The available vacancy in the public (private) sector is given by:

$$V_1(t)(1-\sigma_1)\big(V_2(t)(1-\sigma_2)\big)$$

Where $V_1(t)\big(V_2(t)\big)$ represents the total vacancies available in the public (private sector) and $\sigma_1(\sigma_2)$ accounts for the diminishing rates of these vacancies as a result of the economic downturn. The parameter $\beta$ accounts for the rate of decline in economic activities and $\alpha_1 + \beta(\alpha_2 + \beta)$ accounts for increase in lay-offs as a result of insufficient funds to pay workers during the period of recession. The parameter $b_1(b_2)$ accounts for the rates at which private (public) employed individuals retire from their jobs. Similarly, $C_1\beta\ (C_2\beta)$ accounts for increased rate of forceful retirement. The parameter $K_2$ accounts for the rate at which unemployed individuals in the self-employed class progress to the unemployed class at a rate $K_1 + \beta$ which accounts for increased rate at which businesses are seriously affected and fold up. The parameters $d_1, d_2, d_3, d_4, d_5$ accounts for the death rates in $U(t),\ S(t),\ E_G(t),\ E_P(t)$ *and* $R(t)$ respectively.

Putting all of the above together, we propose below the following model for unemployment under recessions which is described by the system given below:

$$\frac{du}{dt} = \wedge + (\beta + K_1)S + (\alpha_1 + \beta)E_P + (\alpha_2 + \beta)E_G - (K_2 + a_1 v_1(t)(1-\sigma_1) + a_2 v_2(t)(1-\sigma_2) + d_1)U(t)$$

$$\frac{ds}{dt} = K_2\, U(t) + \theta_1 E_P(t) + \theta_2 E_G - (K_1 + \beta + d_2)S(t) + \gamma R$$

$$\frac{dE_P}{dt} = a_1 v_1(t)(1-\sigma_1)\, U(t) - \ (\theta_1 + \alpha_1 + b_1 + \beta + c_1\beta + d_4)E_P(t)$$

$$\frac{dE_G}{dt} = a_2 v_2(t)(1-\sigma_2)\, U(t) - \ (\theta_2 + \alpha_2 + b_2 + \beta + c_2\beta + d_3)E_G(t)$$

$$\frac{dR}{dt} = (b_1 + c_1\beta)\, E_P + (b_2 + c_2\beta)E_G - (\gamma + d_5)R$$

(2)

**Table 1:** Variables and parameters of model (2)

| Variable | Values | Interpretation |
|---|---|---|
| U(t) | 234000 | Unemployed individuals |
| $E_P(t)$ | 208000 | Individuals employed in private sector |
| $E_G(t)$ | 289000 | Individuals employed in public sector |
| R(t) | 12000 | Retired individuals |
| S(t) | 89000 | Self-employed/entrepreneurs |
| $V_1(t)$ | 6000 | Total vacancies available in private sector |
| $V_2(t)$ | 10000 | Total vacancies available in public sector |
| $\beta$ | 0.0158,0.0015 | Rate of decline in economic activities |
| $K_1$ | 0.0015 | Rate at which businesses fold up |
| $K_2$ | 0.07 | Rate of progression to the self-employed class |
| $a_1, a_2$ | 0.0001,0.0008 | Rates at which unemployed individuals are hired in private or public sector |
| $\alpha_1, \alpha_2$ | 0.004, 0.0001 | Rates of lay-offs in the $E_P$ and $E_G$ classes respectively |
| $b_1, b_2$ | 0.05, 0.04 | Rates of retirement in the $E_P$ and $E_G$ classes respectively |
| $\sigma_1, \sigma_2$ | 0.05, 0.04 | Diminishing rate of available vacancies in the $E_P$ and $E_G$ classes respectively due to lack of funds |
| $c_1\beta, c_2\beta$ | 0.00002,0.00001 | Rates of forceful retirement from the $E_P$ and $E_G$ classes respectively. |
| $\gamma$ | 0.0004 | Rate at which those who retire go into business |
| $\theta_1, \theta_2$ | 0.0002, 0.0001 | Rate of progression from the $E_P$ and $E_G$ classes respectively to the self-employed class. |
| $d_i$ (i =1,2<br>0.04,0.001,0.03,<br>,3,4,5) 0.02,0.07 | | Death rates in U(t), (S(t), $E_G(t)$, $E_P(t)$ and $R_{(K)}$ classes respectively |

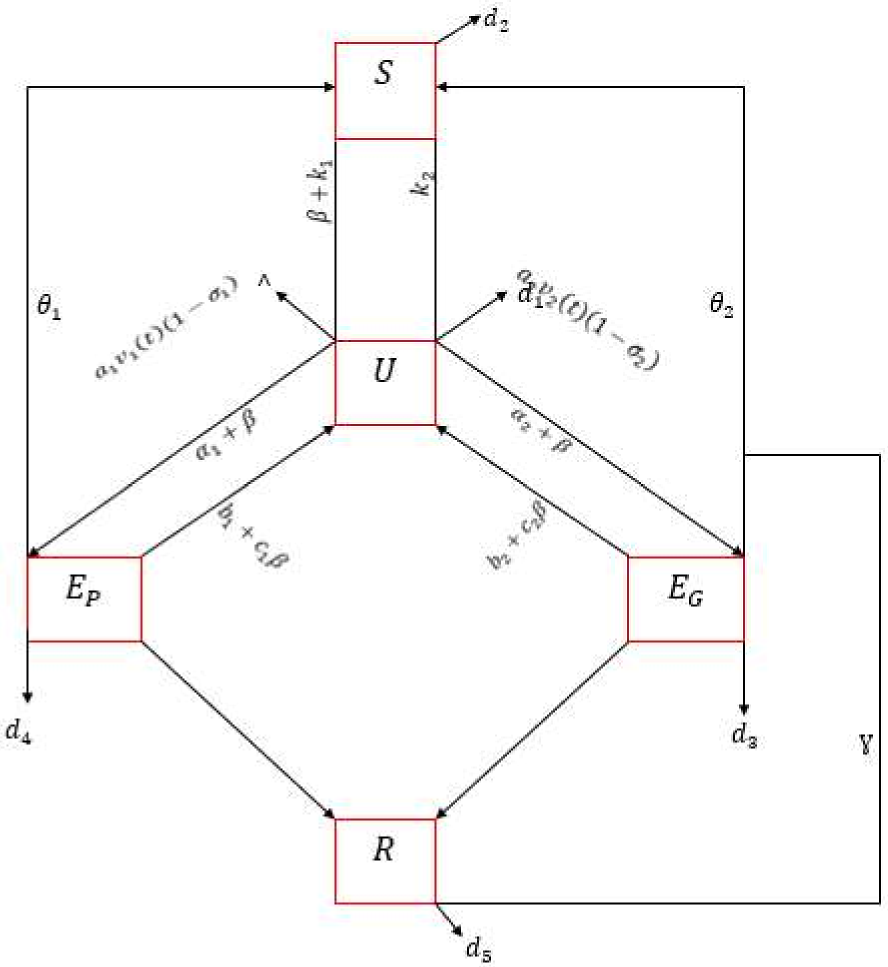


**Fig. 1:** The flow diagram of model (2)

### 2.2 Positivity and Boundedness of Solution

Since model (2) monitors the human population, all associated state variables and parameters are non-negative for all time $t \geq 0$. Thus, the solutions of model (2) with positive initial data will remain positive for all $t \geq 0$. The feasible region of model (2) is given by:

$$\Omega = \left\{(U, E_P, E_G, R, S)\epsilon\mathbb{R}_+^s : 0 \leq U \leq \frac{\wedge}{k_2 + m + d_1}, 0 \leq U + E_P + E_G + R + S \leq \frac{\wedge}{\sum_{i=1}^s d_i}\right\} \quad (3)$$

Where:

$$m = a_1 v_1(t)(1 - \sigma_1) + a_2 v_2(t)(1 - \sigma_2) \quad (4)$$

The next result gives the positivity invariance of (3)

**Theorem 2.1**
The set

$$\Omega = \left\{(U, E_P, E_G, R, S)\epsilon\mathbb{R}_+^s : 0 \leq U \leq \frac{\wedge}{k_2 + m + d_1}, 0 \leq N_H(t) \leq \frac{\wedge}{\sum_{i=1}^s d_i}\right\}$$

is positively invariant and a region of attraction for model (2).

**Proof**
Adding the equations of model (2) gives:

$$\frac{dN_H}{dt} = \wedge - d_1 U(t) - d_2 S(t) - d_3 E_G(t) - d_4 E_P(t) - d_5 R(t) \quad (5)$$

Which gives;

$$\frac{dN_H}{dt} \leq \wedge - dN_H \quad (6)$$

Where

$$d = d_1 + d_2 + d_3 + d_4 + d_5$$

This shows that taking the limit supremum gives:

$$\lim_{t\to\infty} Sup\left(N_H(t)\right) \leq \frac{\wedge}{d} \quad (7)$$

Thus, $N_H(t) \leq \frac{\wedge}{d} \;\; if \;\; N_H(0) \leq \frac{\wedge}{d}$
Hence, Ω is positively invariant. Further, if $N_H(0) > \frac{\wedge}{d}$, then either the solution enters Ω in infinite time or $N_H(t)$ approaches $\frac{\wedge}{d}$. This implies that no solution path leaves through any boundary of Ω and the feasible region Ω attracts all solutions in $\mathbb{R}_+^5$.

### 2.3 Equilibrium Analysis
To complete the equilibrium points of the proposed model, we need to solve the system of differential equations in model (2) at an arbitrary equilibrium denoted by:

$$E_h = (U^*, E_P{}^*, E_G{}^*, R^*, S^*)$$

This gives

$$U^*(t) = \frac{\wedge A_1 A_2(\gamma + d_5)(k_1 + \beta + d_2)}{A_1 A_2(k_2 + m + d_1)(\gamma + d_5)(k_1 + \beta + d_2) - [H(\beta + k_1) + (k_1 + \beta + d_2)(\gamma + d_5)(A_1 M_1(\alpha_1 + \beta) + A_2 M_2(\alpha_2 + \beta))]} \quad (8)$$

$$E_P^*(t) = \frac{a_1 v_1(t)(1 - \sigma_1)U^*(t)}{\theta_1 + \alpha_1 + \beta + d_4} \quad (9)$$

$$E_G^*(t) = \frac{a_2 v_2(t)(1 - \sigma_2)U^*(t)}{\theta_2 + \alpha_2 + \beta + d_3} \quad (10)$$

$$R^*(t) = \frac{(A_1 A_3 M_1 + A_2 A_4 M_2) U^*(t)}{A_1 A_2 (\gamma + d_5)} \tag{11}$$

$$S^*(t) = \frac{[(\gamma + d_5)(A_1 A_2 k_2 + \theta_1 A_1 M_1 + \theta_2 A_2 M_2) + \gamma A_5] U^*(t)}{A_1 A_2 (\gamma + d_5)(k_1 + \beta + d_2)} \tag{12}$$

Where

$M = M_1 + M_2 = a_1 v_1(t)(1 - \sigma_1) + a_2 v_2(t)(1 - \sigma_2)$
$A_1 = \theta_2 + \alpha_2 + b_2 + \beta + c_2\beta + d_3$
$A_2 = \theta_1 + \alpha_1 + b_1 + \beta + c_1\beta + d_4$
$A_3 = b_1 + c_1\beta$
$A_4 = b_2 + c_2\beta$
$A_5 = A_1 A_3 M_1 + A_2 A_4 M_2$
$H = (\gamma + d_5)(A_1 A_2 k_2 + \theta_1 A_1 M_1 + \theta_2 A_2 M_2) + \gamma(A_1 A_3 M_1 + A_2 A_4 M_2)$

All the parameters from (6) – (9) are strictly positive. This implies that the numerators of (6) – (9) are always positive since

$$M_1 = a_1 v_1(t)(1 - \sigma_1) \;\; and \;\; M_2 = a_2 v_2(t)(1 - \sigma_2), \;\; 0 < \sigma_1 < 1, \quad 0 < \sigma_2 < 1$$

Equations (6) – (9) are dependent on $U^*(t)$, hence we consider the unemployment equilibrium point $U^*(t)$.

**2.4 Effect of Some Parameters on the Equilibrium Level of the Unemployed Population**

We analyze the effects of economic decline $(\beta)$ rates of progression from the unemployed class to the $E_P$ and $E_G$ classes $(m = m_1 + m_2)$ as well as the rates of progression $(k_1, k_2)$ on the equilibrium level of the unemployed population.
First, to analyze the effect of $\beta$ on the unemployed population, we differentiate the equilibrium point $U^*(t)$ with respect to $\beta$. We obtain:

$$\frac{\partial u^*}{\partial \beta} = \frac{\wedge A_1 A_2 (\gamma + d_5)[H d_2 + (\gamma + d_5)(k_1 + \beta + d_2)^2 (A_1 m_1 + A_2 m_2)]}{[A_1 A_2 (k_2 + m + d_1)(\gamma + d_5)(k_1 + \beta + d_2) - [H(\beta + k_1) + (k_1 + \beta + d_2)(\gamma + d_5)(A_1 m_1 (\alpha_1 + \beta) + A_2 m_2 (\alpha_2 + \beta))]]^2} \tag{13}$$

From (13), we have that:

$$\frac{\partial u^*}{\partial \beta} > 0$$

This shows that the equilibrium level of the number of unemployed individual is an increasing function of $\beta$ (the equilibrium level of the number of unemployed individuals increases as the rate of economic decline increases. This signifies a detrimental impact on the economy as there will be a huge burden of unemployment in the population which will adversely affect the economy.
We now turn to analyze the impact of the rate of progression from the unemployed class to the $E_P$ and $E_G$ classes $(m = m_1 + m_2)$ on the equilibrium level of the unemployed population. The result shows that:

$$\frac{\partial u^*}{\partial m} = \frac{-\wedge A_1^2 A_2^2 (k_2 + d_1)(\gamma + d_5)(k_1 + \beta + d_2)^2}{[A_1 A_2 (k_2 + m + d_1)(\gamma + d_5)(k_1 + \beta + d_2) - [H(\beta + k_1) + (k_1 + \beta + d_2)(\gamma + d_5)(A_1 m_1 (\alpha_1 + \beta) + A_2 m_2 (\alpha_2 + \beta))]]^2} \tag{14}$$

The result reveals that a rapid progression from the unemployed class to the $E_P$ and $E_G$ classes will have a positive impact on unemployment control in Nigeria population. This rapid progression will depend on the state of the economy, vacancies created in the public and private sectors, rates of layoffs and other factors.
Evaluation of the impact of the progression rates $k_1$ and $k_2$ reveals that:

$$\frac{\partial U^*}{\partial k_1} = \frac{\wedge A_1 A_2 d_2 H(\gamma + d_5)}{[A_1 A_2 (k_2 + m + d_1)(\gamma + d_5)(k_1 + \beta + d_2) - [H(\beta + k_1) + (k_1 + \beta + d_2)(\gamma + d_5)(A_1 m_1 (\alpha_1 + \beta) + A_2 m_2 (\alpha_2 + \beta))]]^2} \tag{15}$$

and

$$\frac{\partial U^*}{\partial k_2} = \frac{-\wedge A_1 A_2 d_2 (\text{y}+d_5)(k_1+\beta+d_2)}{\left[A_1 A_2 (k_2+m+d_1)(\text{y}+d_5)(k_1+\beta+d_2) - \left[H(\beta+k_1)+(k_1+\beta+d_2)(\text{y}+d_5)\left(A_1 m_1(\alpha_1+\beta)+A_2 m_2(\alpha_2+\beta)\right)\right]\right]^2} \quad (16)$$

Hence, the equilibrium level of the number of unemployed individuals is an increasing function of $k_1$. This signifies a negative impact on the economy as the burden of unemployment will increase. Whereas a high progression rate from the unemployed class to the self employed class will have a positive impact in reducing the burden of unemployment in the population. The results obtained are summarized below.

**Lemma 2.1:** A rapid progression from the unemployed class to the $E_P, E_G$ and $S$ classes will have:

1. A positive impact on unemployment control in the population if $k_2 > k_1 + \beta$ and $M > \alpha_1 + \alpha_2 + \beta$
2. No impact on unemployment control in the population if $k_2 = k_1 + \beta$ and $M = \alpha_1 + \alpha_2 + \beta$
3. A detrimental impact on unemployment control in the population if $k_2 < k_1 + \beta$ and $M < \alpha_1 + \alpha_2 + \beta$

In the analysis above, a rapid progression refers to a high progression rate from one class to another. The rapid progressions from the unemployed class depends on the state of the economy, environmental factors, access to capital for business start up, available vacancies in public and private sectors of the economy as well as other factors. The mathematical relations established in (13) – (16) shows how the crucial parameters $(\beta, m, k_1, k_2)$ can be harnessed to provide effective control of unemployment in a recessed economy.

### 2.5 Stability Analysis

From the result obtained, we now study local stability of the equilibrium:

$$E_h = (U^*, E_P^*, E_G^*, R^*, S^*)$$

To obtain this, we compute the Jacobian (variational matrix) J of model (2)

$$J = \begin{bmatrix} -(k_2+m+d_1) & \beta+k_1 & \alpha_1+\beta & \alpha_2+\beta & 0 \\ k_2 & -(k_1+\beta+d_2) & \theta_1 & \theta_2 & \text{y} \\ m_1 & 0 & -A_2 & 0 & 0 \\ m_2 & 0 & 0 & -A_1 & 0 \\ 0 & 0 & A_3 & A_4 & \text{y}+d_5 \end{bmatrix} \quad (17)$$

The characteristic equation associated with the variational matrix above is given by:

$$\lambda^5 + \beta_0\lambda^4 + \beta_1\lambda^3 + \beta_2\lambda^2 + \beta_3\lambda + \beta_4 = 0 \quad (18)$$

Where

$$\beta_0 = (\text{y}+d_5) + A_1 + A_2 + (k_2+m+d_1) + (k_1+\beta+d_2)$$

$$\beta_1 = (\text{y}+d_5) + [A_1 + A_2 + (k_2+m+d_1) + (k_1+\beta+d_2)] + A_1[A_2 + (k_1+m+d_1) + (k_1+\beta+d_2)] + (k_2+m+d_1)\left((k_1+\beta+d_2)+1\right) + (k_1+\beta+d_2)$$

$$\beta_2 = (\text{y}+d_5) + \left[A_1\left(A_2 + (k_2+m+d_1) + (k_1+\beta+d_2)\right) + (k_2+m+d_1)(k_1+\beta+d_2) + 1 + (k_1+\beta+d_2)\right] + (k_2+m+d_1)(k_1+\beta+d_2)(A_1+A_2) + A_1\left((k_1+\beta+d_2)+(k_2+m+d_1)\right)$$

$$\beta_3 = (\text{y}+d_5)\left[A_1\left(k_2+m+d_1(k_1+\beta+d_2+1)+(k_1+\beta+d_2)\right)\right] + A_2(k_2+m+d_1) + (k_1+\beta+d_2)[\text{y}+d_5+A_1]$$

$$\beta_4 = A_1 A_2 (\text{y}+d_5)(k_1+\beta+d_2)(k_2+m+d_1)$$

From the characteristic polynomial obtained, we obtain the following result:

**Lemma 2.2:** The equilibrium $E_h = (U^*, S^*, E_P^*, E_G^*, R^*)$ is locally asymptotically stable without any condition.

**Proof:** From the result obtained in (18), the constants $B_0, B_1, B_2, B_3, and\ B_4$ are positive. Now using the Routh-Hurwitz criterion for a fifth degree polynomial implies that (18) has all the roots in the left half plane if and only if all coefficients of (18) are positive (Dorf and Bishop, 2001). Thus, we can conclude that the roots of (18) are either negative or have negative real parts.

**Theorem 2.3:** The equilibrium $E_h$ given by (8) to (12) is globally asymptotically stable.

**Proof**
We define a new dependent variable called a Lyapunov function of the form:

$$L(t) = A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)U(t) + (\text{y} + d_5)(k_2 + m + d_1)S(t)$$

So that:

$$\frac{dL}{dt} = A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)U(du) + (\text{y} + d_5)(k_2 + m + d_1)S(ds)$$

$$= A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)U(\wedge + (\beta + k_1)S + (\alpha_1 + \beta)E_P + (\alpha_2 + \beta)E_G - (k_2 + m + d_1)U) + (\text{y} + d_5)(k_2 + m + d_1)S(k_2U + \theta_1E_P + \theta_2E_G - (k_1 + \beta + d_2)S)$$

$$\frac{dL}{dt} = -A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)[(\wedge + (\beta + k_1)S + (\alpha_1 + \beta)E_P + (\alpha_2 + \beta)E_G - (k_2 + m + d_1)U]$$

$$-(\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)[k_2U + \theta_1E_P + \theta_2E_G - (k_1 + \beta + d_2)S]$$

$$\frac{dL}{dt} = -A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\wedge - A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(\beta + k_1)S$$

$$-A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)(\alpha_1 + \beta)E_P - A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(\alpha_1 + \beta)E_G$$

$$+A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)^2U - (\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)k_2U$$

$$-(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\theta_1E_P - \theta_2(\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)E_G$$

$$+(\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)^2S$$

$\frac{dL}{dt}$ = X – Y,

Where,

$$X = A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)^2U + (\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)^2S$$

$$\text{Y} = A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\wedge + A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(\beta + k_1)S$$

$$+A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)(\alpha_1 + \beta)E_P + A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(\alpha_1 + \beta)E_G$$

$$(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\theta_1E_P + \theta_2(\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)E_G$$

$$(\text{y} + d_5)(k_2 + m + d_1)(k_1 + \beta + d_2)k_2U.$$

For $U = S = E_P = E_G = 0$

$$\frac{dL}{dt} = -A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\wedge$$

Clearly, if X < Y, then $\frac{dL}{dt} \leq 0$. Also, $\frac{dL}{dt} < 0$ if $U = S = E_P = E_G = 0$, since $A_1A_2(\text{y} + d_5)(k_1 + \beta + d_2)(k_2 + m + d_1)\wedge < 0$. Thus the equilibrium $E_h$ is globally asymptotically stable if X < Y and the proof is complete.

## 3. Numerical Simulation
In this section we simulate the outcome of model (2) choosing parameters informed by economic literature. We present result illustrating the behavior of the system introduced theoretically. We compare the outcome of our result for two values of the parameter β; 0.0158 (1.58%) and 0.00158 (0.158%). The first value of the parameter β indicates a higher rate of economic decline compared to the second value. The values for other parameters are shown in table 1.
First, we simulate the situation where the rate of economic decline β = 0.0158 (1.58%).

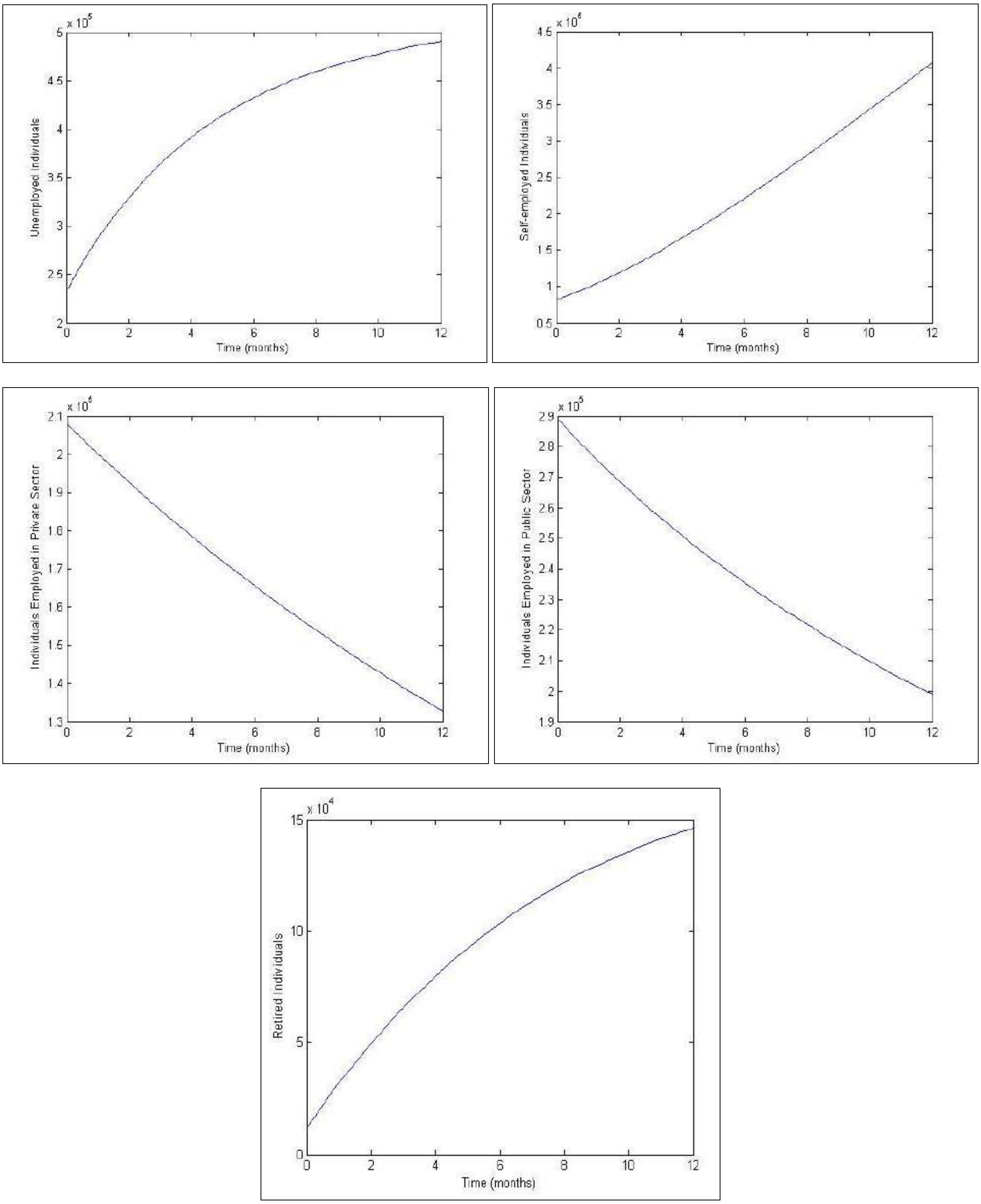


**Fig 2:** Simulation of model (2) with β = 0.0158.

Figure 2 reveals an increase in the rate of unemployment due to the rate of economic decline. It reveals an increase from $2.34 \times 10^4$ to over $4.5 \times 10^5$. This implies that during the period of recession, a number of employed persons lose their jobs and new entrants into the labor force are unable to find jobs as job vacancies as stalled during this period. Similarly, the results reveal an increase in the number of individuals who are self-employed. This is attributable to the frustration encountered during this period as those who are laid-off, new entrants and even the retired individuals might decide to start up their own businesses. The result also reveals a decline in the $E_P$ class from $2.08 \times 10^5$ to a value below $1.4 \times 10^5$ which indicates massive lay-offs during this period. The $E_G$ class is not left out as the is a decline in the number of individuals employed in public sectors from $2.89 \times 10^5$ to $2 \times 10^5$. From the result obtained, it is evident that the rate of economic decline tends to affect all classes of the population.

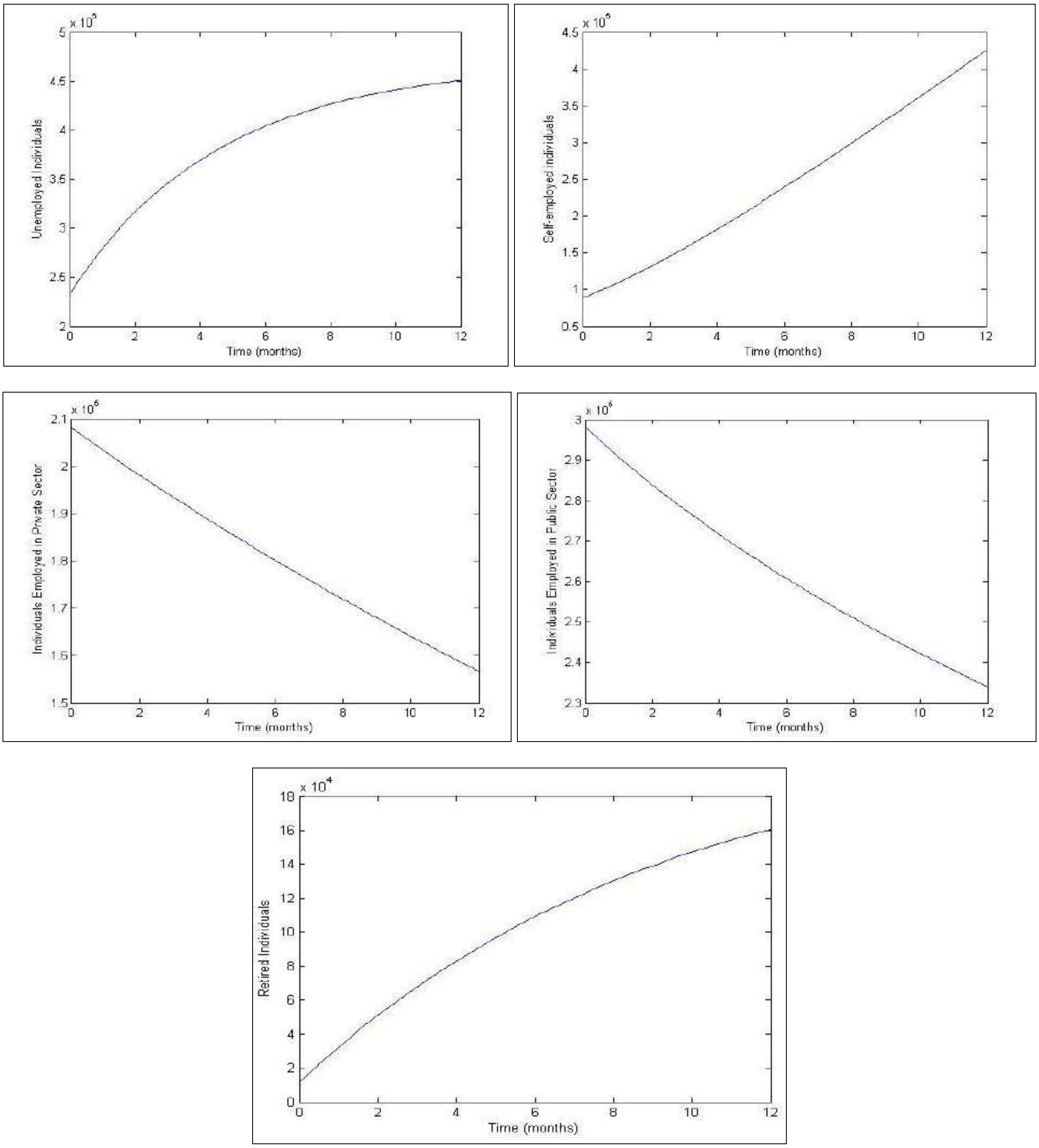


**Fig 3:** Simulation of model (2) with $\beta$ = 0.00158

Simulating model (2) with β = 0.00158 (0.158%) revealed that the number of unemployed individuals increased to $4.5 \times 10^5$ which is lower than the value obtained from figure 1. This implies that the rate of economic decline determines the rate of increase in the unemployed population. Similarly, the rate of decline in the $E_P$ and $E_G$ lower as seen in figure 1. The implication of this is that a higher rate of economic decline connotes a higher number of individuals in the unemployed class.
We notice that the numerical simulations in figure 1 and 2 verify the theoretical findings in section 2.

## 4. Conclusion

A new mathematical model for unemployment in a recessed economy is designed and used to assess the impact of the rate of economic decline on unemployment. Numerical simulation of the model was done to assess the impact of economic decline on unemployment in the population. Simulations of our model corresponded with the theoretical findings. During the period of recession, the rate of unemployment increases depending on the rate of economic decline. This is due to the fact that new entrants into the labor force will be unable to find jobs due to unavailable vacancies in government and private sectors of the economy, government and private institutions tend to lay-off workers so as to reduce cost of operation, some individuals might be forcefully retired in a bid to reduce the number of paid workers. The rate of lay-offs is dependent on the rate of economic decline. from our numerical results, an increased rate of economic decline (1.58%) results in an increase in the number of unemployed individuals. During this period, the number of self-employed individuals also increases, hence government strategy should be focused on developing Small and Medium Scale Enterprises (SMEs) so that a greater percentage of new entrants and even those laid-off would have ample opportunity to start up their own businesses thereby developing themselves and even creating job opportunities for others.

As future work, we plan to consider the effect of skill acquisition on the dynamics of unemployment in a recessed economy. For that, we may decide to include the skill class which represents individuals who have developed one form of skill or the other. The two control classes we would have (self-employed and skilled class) will present different government concerns: providing avenues for skill development and providing financial resources to the self-employed class for transition to the self-employed class instead on depending on the government or relevant private institutions to provide jobs.

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